

Homological 予想・BCM 代数

(可換環論サ2-ス7-12
at 東工大 (8/26)

§1 Homological conjecture

--- (Noetherian) local ring 上の (f, g) module の

不変量 の homological な解釈に基づいて

一連の予想

e.g. Krull 次元 $\leftrightarrow$ homological 次元

(proj, inj, global ...)

Recall $(A, \mathfrak{m}, k)$: local ring に $\neq 1$? 標数は $4/1/0-2$

	char A	char k
equiv. char 0	0	0
equiv. char p	p	p
mixed char (0, p)	0	p
mixed char (p^n , p)	p^n	p

A : equiv. char

$(\Rightarrow) \exists \varphi: A \rightarrow k: \varphi \in A$

A : domain

$\Rightarrow$ char A is 0 or prime

Note

$\text{mod } A$: Category of f.g. A -modules

$\text{Mod } A$: Category of A -modules

1960 ~ 1970 前年

Serre, Auslander, Peskin, Szapiro... e.t.c.
代表: (new) intersection thm.

thm (Serre, 1965, intersection thm)

A : regular local. $M, N \in \text{mod } A$, $\ell(M \otimes_A N) < \infty$
then $\dim M + \dim N \leq \dim A$

(why intersection? $\rightarrow$ Har, Chap I, Prop. 7.1)

Recall Auslander - Buchsbaum formula.

(A m): Noe. loc. $M \neq 0 \in \text{mod } A$, $\text{prj. dim } M < \infty$
 $\text{prj. dim } M + \text{depth } M = \text{depth } A$

A-B formula + intersection thm

$$\begin{aligned}\rightarrow \text{prj. dim } M &= \text{depth } A - \text{depth } M \\ &= \dim A - \text{depth } M\end{aligned}$$

$$\geq \dim A - \dim M \geq \dim N \quad \text{etc.}$$

$\rightarrow$ intersection conj.

$(A, \mathfrak{m})$: Noe. local. $M \neq 0$, $N \in \text{mod } A$ $\ell(M \otimes N) < \infty$

$$\text{then } \dim N \leq \text{prj. dim } M$$

(何で? (u?)

$$\left(\begin{array}{l} \ell(M) < \infty, \text{prj. dim } M < \infty \text{ etc } \dim A \leq \text{prj. dim } M \\ \leq \dim A \end{array} \right)$$

$\rightarrow$ new intersection thm (NIT)

$(A, \mathfrak{m})$: Noe. loc.

$$F_\bullet: 0 \rightarrow F_m \rightarrow \dots \rightarrow F_0 \rightarrow 0: \text{not exact}$$

F_i : fin. free

$$0 \leq i \leq m, \ell(H_i(F_\bullet)) < \infty \Rightarrow \dim A \leq m$$

equiv. char $p > 0$ and $\neq$.

Frobenius functor $\mathbb{F}$ is $\mathbb{F}^2$ is $\mathbb{F}$.

A: char $p > 0$.

$F: A \rightarrow A: a \mapsto a^p$: Frobenius

A^F : 両側 A -module $\mathbb{F}$. $A^F = A$ (as set)

$$a \in A, r \in A^F, a \cdot r = ar = ra, \quad ra = ra^p$$

すると $f: \text{Mod}_L(A) \rightarrow \text{Mod}_L(A): M \mapsto A^F \otimes_A M$

これは $\mathbb{F}$ Frobenius functor である.

$\mathbb{F}$ は free module $\mathbb{F}$ 保ち, localisation と可換.

Sketch (equiv. char $p > 0$ and $\neq$)

A: complete $\mathbb{F}$ and $\mathbb{F}$.

まず $\varphi_1(F_1) \subset mF_0$ and $\mathbb{F}$ (technical fix easy)

$\mathbb{F}(F_0)$ は $\varphi_i = (a_{i,j})$ and $\mathbb{F}$,

$(a_{i,j}^p) \in \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{F}$.

and $\forall p \in \text{Spec } A \setminus \{m\}$ is $\mathbb{F} \otimes A_p$ is exact

$\mathbb{F}(A, m)$: local $M \in \text{mod } A$

$$\ell(M) < \infty \iff \text{Supp } M = \{m\}$$

$$\forall i, H_i(\mathcal{F}(F_0))_p = 0$$

$$\text{or } \ell(H_i(\mathcal{F}(F_0))) < \infty$$

$$\text{or } \mathcal{F}(\varphi_1)(F_1) \subset \mathfrak{m}^p F_0 \text{ for all } p$$

$$\text{or } \mathcal{F}^e(\varphi_1)(F_1) \subset \mathfrak{m}^{pe} F_0 \text{ \& Homology has fin. length.}$$

$$\text{or } I_i := \bigcap_{n \geq i} H_n^i(A) \text{ s.t. } I_0 \cdots I_n \subset \bigcap_{n \geq 0} H_0(\mathcal{F}^e(F_0))$$

$\Rightarrow F_0/\mathfrak{m}F_0$
(by Roberts's lemma)

$$\text{or } I_0 \cdots I_n \subset \mathfrak{m}^{pe}$$

$$\text{or } I_0 \cdots I_n \subset \bigcap_{e \geq 1} \mathfrak{m}^{pe} = 0$$

$$\text{or } I_0 \cdots I_n \text{ is not a primary ideal.} \quad \square$$

§2 DSC & MC

thm (Direct Summand Conjecture. DSC)

A : reg. loc. $A \subset B$: A -alg. f.g. as A -mod

$\Rightarrow B$ is A -module (i.e. $A \in \text{直和因子}$)

cf $I \subset A$: ideal $\neq 0, \neq A$

① $A \rightarrow B$: faithfully flat

or

② $A \subset B$: 直和因子 (i.e. $A \hookrightarrow B$: split)

$\Rightarrow IB \cap A = I$

① $\dots \forall M \in \text{Mod } A, M \rightarrow M \otimes B: x \mapsto x \otimes 1$ is inj. f.a.c.

$A/I \rightarrow B/IB$ is inj.

② $\dots B = A \oplus C$ (i.e. $IB = I \oplus IC$) $IB \cap A = I$

Obs $(A, n), (B, n)$: Noe. loc. $A \subset B$. f.g. A -mod

$p' \in \text{Spec } B, mB \subset p' \subset n$ (i.e. $p' \in \text{Spec } B$)

$A \subset B$: integral f.a.c. $p' = n$ (lying over thm)

f.a.c. q : s.o.p. of A is B a s.o.p. $\neq q$

Obs

$B: CM \wedge \exists z. DSC \text{ of } \exists z \perp z$

$\therefore) A \subset B: \text{integral } \exists z \perp \dim A = \dim B$

$g: \text{S.O.P. of } A \exists z \exists z. B \text{ a S.O.P. } z \notin \mathbb{A} \exists z.$

$B: (CM \text{ of}) \quad g: B \text{ regular.}$

$\exists z \dim A \leq \text{depth}_A B = \text{depth } B = d.$

$A: \text{reg. of} \quad \text{prj. } \dim_A B < \infty.$

$A-B \text{ formula of}$

$\text{prj. } \dim_A B + \text{depth } B = \text{depth } A$

$\rightarrow \text{prj. } \dim_A B = 0.$

$B: A\text{-flat} (\Rightarrow) B: A\text{-free } \exists z \perp z. B: A\text{-free} //$

thm (monomial conjecture MC)

(A.m): Noe. loc. $\underline{a} = a_1, \dots, a_r$ s.o.p.

$$\forall t > 0, a_1^t \dots a_r^t \notin (a_1^{t+1}, \dots, a_r^{t+1})$$

- s.o.p. は "独立, $\neq 0$ " 3つ 3つ と主張してる。

$$(X_1, \dots, X_r \text{ a.f.})$$

- $(DSC \Leftrightarrow MC) \Rightarrow \text{NIT (new-intersection thm)}$

thm 1

(A.m): reg. loc. $\underline{a}$: reg. s.o.p.

$A \subset B$: B : A -alg, f.g. as A -mod.

A : direct summand of B

$$\Leftrightarrow \forall t > 0, a_1^t \dots a_r^t \notin (a_1^{t+1}, \dots, a_r^{t+1})B$$

Proof

$$(\Rightarrow) \quad \underline{a} : \text{reg. f.a.r. } a_1^t \dots a_r^t \notin (a_1^{t+1}, \dots, a_r^{t+1})$$

$$\forall I \subset A: \text{ideal } 1 = \sum I B \cap A = I \cdot 1$$

$$a_1^t \dots a_r^t \notin (a_1^{t+1}, \dots, a_r^{t+1})B$$

($\Leftarrow$) A : complete & z su

$\forall t > 0, I_t := (a_1^t, \dots, a_n^t), A_t := A/I_t, B_t := B/I_t B$

u# $A \rightarrow B$ かつ 等かた子 $A_t \xrightarrow{p_t} B_t$ は inj.

($\Rightarrow$) technical. A_t : 0-dim Henselstein (= $\mathbb{Z}_p$),
 $a_1^t \dots a_n^t + I_t \in \text{Soc } A_t \subset \ker(p_t \downarrow)$

u# A_t : inj. A_t -module $\neq \emptyset$

$D_t := \{ \varphi \in \text{Hom}(B_t, A_t) \mid \varphi \circ p_t = \text{id}_{A_t} \} \neq \emptyset$

すなわち $(D_t)_t$ は Mittag-leffler な

集合の列に存在.

$$\begin{array}{ccc} & \text{id} \nearrow & A_t \\ & & \uparrow p_t \\ A_t & \hookrightarrow & B_t \end{array}$$

(X_i, φ_{ij}) : Mittag-leffler
 $\stackrel{\text{def}}{\Leftrightarrow} \forall i, \exists j \geq i: \forall k \geq j, \varphi_{ki}(X_k) = \varphi_{ji}(X_j)$

(X_i, φ_{ij}) : Mittag-leffler & I -countable & $\forall i, X_i \neq \emptyset$

$$\Rightarrow \varprojlim X_i \neq \emptyset$$

すなわち $\varprojlim D_t \neq \emptyset$ i.e. $A \rightarrow B$ は Split //

Thm 2

$(A, m) : \text{Noe. loc. } \mathfrak{a} = a_1, \dots, a_r : \text{s.o.p. of } A.$

$\exists M \in \text{Mod } A : \mathfrak{a} = M\text{-reg} \Rightarrow \forall e > 0, a_1^e, \dots, a_r^e \notin (a_1^{e+1}, \dots)$

$\therefore a_1^e, \dots, a_r^e \in (a_1^{e+1}, \dots)$ LTB.

$\exists \mathfrak{a}^e : a_1^e, \dots, a_r^e \in M \subset (a_1^{e+1}, \dots) M^e, I := (a_1, \dots, a_r) \subset \mathfrak{a}^e$

$\text{gr}_I(M) \cong M \otimes A_I[X_1, \dots, X_r] \quad (\leftarrow \text{regular } \mathfrak{a} := (1, 1, \dots))$

$X_1^e, \dots, X_r^e \in \text{gr}_I(M) \subset (X_1^{e+1}, \dots, X_r^{e+1}) \text{gr}_I(M)$

LTB

$\text{gr}_I(M) / (X_1^{e+1}, \dots) \text{gr}_I(M) \cong \bigoplus_{X_1^{e_1}, \dots, X_r^{e_r} \notin (X_1^{e+1}, \dots)} X_1^{e_1} \dots X_r^{e_r} (M/I^n) \downarrow$

L

$\cap M = \{ \nabla \}$

§3 big CM conjecture.

Def

$(A, m) : \text{Noe. loc. } M \in \text{Mod } A$

$\underline{g} : \text{S.o.p. } 1 \leq i \leq r, \underline{g} : M\text{-reg. } 1 \leq i \leq r,$
 $M \in \underline{g} \text{ is a big CM module.}$

thm (André, 2016)

$(A, m) : \text{Noe. loc. } \forall \underline{g} : \text{S.o.p.}$

$\exists M \in \text{Mod } A : \underline{g} : M\text{-reg.}$

e.g.

$\psi : \text{fld. } A := \mathbb{K}[[X, Y]], \mathbb{K}[[X, Y]] = A/(Y) : A\text{-mod}$

$M := A \oplus \text{Frac}(\mathbb{K}[[X, Y]])$ is a big CM module.

X, Y is a big CM module. For Y, X is a big CM module.

For M is a big CM module.

Def

$\forall \mathfrak{a} = \text{s.o.p. } \mathfrak{a} : M\text{-reg.} \iff \exists \text{ } M \in$

balanced big CM ring

Def

$\mathfrak{a} = a_1, \dots, a_r \in A, \quad I := (\mathfrak{a})$

$M \neq \mathbb{Z}M, \forall f \in M[x_1, \dots, x_r] : \nexists$

$f(\mathfrak{a}) \in I^{\deg f + 1}M \Rightarrow \forall (\text{coeff. of } f) \in \mathbb{Z}M$
 $\nexists \text{ } \mathfrak{a} \in \text{quasi-}M\text{-reg. ring}$

Thm (Rees)

$\mathfrak{a} : \text{reg} \Rightarrow \mathfrak{a} : \text{f-reg}$

(松村 16.2)

Prop

$\hat{M} : M \text{ a } \mathbb{Z}\text{-a completion}$

TF A E :

(i) $\mathfrak{a} : M\text{-f-reg}$

(ii) $\mathfrak{a} : \hat{M}\text{-f-reg}$

(iii) $\mathfrak{a} : \hat{M}\text{-reg}$

(Bruns-Herzog Thm 8.5.1)

thus

(A, m): Noe. loc. g: s.o.p. M: g-big CM

M -adic completion is balanced by CM .

c) $r := \dim A \neq 1$ (I-adic と m-adic は同じに注意)

b : b_1, b_r S.O.P. 473.

$$\exists c \in A : a_1, \dots, a_{r-1}, c \in S_{0,p}$$

2

$b_1 \rightarrow b_m$, C: S.O.P (using Prime avoidance)

$u \neq a_1, \dots, a_{r-1}$, C は M -reg. (≥ 2) ($\frac{D}{\text{in}}(Z_1) \cap Z_2 \cap L \cap C$...)

$$\hat{M} - \text{neg. l.} \Rightarrow (M - \text{neg.} \Rightarrow M - \delta - \text{neg.} \Rightarrow \hat{M} - \text{neg.})$$
$$\mathfrak{g} \in (c, a_1, \dots, a_{r-1}) : \hat{M}\text{-reg.}$$

また $(1 - \sigma - \varepsilon) \tau_1, \dots, \tau_{r-1} \in A/\mathfrak{c}A$ は $\hat{A}/\mathfrak{c}\hat{A}$ -reg. c.

$$C, b_1, \dots, b_n \text{ ist } \mu\text{-mg}$$
$$b_1, \dots, b_{r-1}, c : A\text{-} \text{mg.}$$
$$b_1, \dots, b_{r-1}, b_r, \hat{A} - \text{neg.}$$

big CM $\Rightarrow$ DSC a sketch

lem 3

A : domain $A \subset B$: integral ext.

$P' \in \text{Spec } B$: $P' \cap A = (0)$,

$A \nmid B/P'$ a 直和因子 $\Rightarrow A \nmid B$ a 直和因子

$$\begin{array}{ccc} \text{ii)} & A & \hookrightarrow B \\ & \searrow & \downarrow \\ & & B/P' \end{array}$$

Def

$(A, \mathfrak{m}, k)$: Henselian local

$\stackrel{\text{def}}{=} \forall f, g_0, h_0 \in A[x]$: monic.

$f = g_0 h_0 \in \mathfrak{m} A[x]$ & $(g_0, h_0) + \mathfrak{m} A[x] = A[x]$

$\Rightarrow \exists g, h \in A[x]$: monic:

$f = gh$ & $g - g_0, h - h_0 \in \mathfrak{m} A[x]$

lem 4

(A.m.k): Henselian local

$A \subset B$: integral ext. B : domain $\Rightarrow B$: local

$$\therefore) u+u' \in \text{Spec } B, \quad b \in u \quad b \notin u' \quad \text{q.e.d.}$$
$$A \subset B: \text{int. ext.} \leadsto \exists f = X^n + a_1 X^{n-1} + \dots + a_n \in A[X];$$
$$f(b) = 0.$$
$$\leadsto a_n = -(y^n + \dots + a_{n-1}y) \in \mathfrak{A} \cap A = m_{f(y)}$$

$\{ \langle i, a_i \rangle \in m \mid a_i \in m \wedge \exists n \in \mathbb{N} (A = m \wedge \langle b^n \rangle \in \mathcal{N} \wedge \langle a_i \rangle \in \mathcal{N} \wedge \langle a_i \rangle \in \mathcal{N} \wedge \langle a_i \rangle \in \mathcal{N})$

$$\exists i: a_i \notin m.$$
$$a_n, \dots, a_{n-(l-1)} \in m, \quad a_{n-l} \notin m \text{ and } l \leq l_0.$$

for $\psi(x)$ and $\psi(x)$

$$X^n + \dots + a_{n-1}X^1 = X^1 (\quad)$$

(6) A: Henselian fact: $\exists g, h \in A[x]$, monic,

$$f = gh, B: \text{int. dom. } \mathbb{R}, \quad g(b) = 0 \text{ or } h(b) = 0$$

c) かつ λ の次数が真に ∞ なら, $b \in A$ となり ∞

big (M $\Rightarrow$) DSC n Sketch

A: complete 4/2 ju.

lem 35) B_i domain, lem 4 51) B_i local 125.

9: neg. s.o.p. of A $\nexists x \exists y$ s.t. B is s.o.p. of $\neg \exists x \exists y$.

for $\forall n \in \mathbb{N}$, $t_n > 0$, $a_1^{t_n} \dots a_r^{t_n} \notin (a_1^{t_{n+1}}, \dots, a_r^{t_{n+1}})B$

the 1st A is a π -bond and σ -bond //

Hochster is equiv. char $p > 0$ & big CM r.f.l.

2. 1. "Meta theorem" $\in \mathcal{P}$, $\geq$ equi. char $p=0$ $\exists \bar{t} \in \mathcal{C}$.

まず, modification を定数とする。

$\underline{a} = a_1, \dots, a_r$ for M -regular $\mathbb{Z}$.

$$0 \leq s \leq r-1 \quad a_{s+1} : M/(a_1, \dots, a_s)M - kq$$

そうでないとき、それを補正して、というアイデア

Def

A ring. $\underline{a} = a_1, \dots, a_r \in A$. $M \in \text{Mod } A$. $0 \leq s < r$

$e_1, \dots, e_s$ (standard) basis of A^s

$\exists y \in M: a_{s+1} y \in (a_1, \dots, a_s)M \nleftrightarrow \exists$

$M' := M \oplus A^s / A_w$, $w := y - \sum_{i=1}^s a_i e_i \nleftrightarrow \exists$

$\perp$ $M' \in \text{Ma}(\exists \perp \text{Mod } A)$ $\underline{a}$ -modification of types $\in \mathbb{N}$

$x \in M \nleftrightarrow \exists \nexists$. $\exists N \in \text{Mod } A: \exists y \in N$

$\exists f: M \rightarrow N: f(x) = y \nleftrightarrow \exists \exists \nexists$

$f: (M, x) \rightarrow (N, y) \nleftrightarrow \exists$

$(M, x) \rightarrow (M_1, x_1) \rightarrow \dots (M_g, x_g) \nleftrightarrow$

$(M_{i+1}, x_{i+1}) \text{ or } (M_i, x_i) \cap S_{i+1} - \underline{a}$ -modif. $\nleftrightarrow$

$\nexists \exists \exists \exists \exists \nexists$, = a sequence $\in \text{Ma}$

$(S_1, \dots, S_g) - \underline{a}$ -modification sequence $\in \mathbb{N}$

$(M_g, x_g) \in \text{Ma}(S_1, \dots, S_g) - \text{modif. } \in \mathbb{N}$

Def

$(M, \alpha) \models \exists x, z, x \neq z \wedge \alpha x \neq \alpha z$ $(M, \alpha) \models$

non-degenerate α

Prop 5

A : Noetherian $\alpha = \alpha_1, \dots, \alpha_r \in A$

TF $A \models$

(i) $\exists M \in \text{Mod } A: \alpha: M \text{-regular}$

(ii) $(A, \alpha) \wedge \alpha$ -modif $(M, \alpha) \models \exists x, z$ non-degenerate

(technical)

$\Rightarrow$ for big CM rings $\exists$ equiv. char $p > 0$ or $\bar{\alpha} \neq \alpha$

Sketch (equiv. char $p > 0$ is $\neq 1$ big CM conj)

A : complete & i.c.c.

Prop 5 f) $(N, \gamma) \in (A, I)$ a $\underline{g}$ -modif & $I \nsubseteq \mathbb{Z}$

non-deg. $\exists \bar{x} \in I^{\vee}$ s.t.

$(N, \gamma) \in (A, I)$ a $(S_1, \dots, S_r)_{\underline{g}}$ -modif. $\bar{x}$ deg. & $\nabla \bar{z}$.

$\forall e \geq 1, (f^e(N), f^e(\gamma))$ is (A, I) a $\underline{g}^{pe}$ -modif.

if $\exists c \in A$: non-nilpotent; $\forall e,$

$$(A, I) \rightarrow (f^e(N), f^e(\gamma))$$

$$\searrow \quad \downarrow \exists \varphi_e$$

$$(A, c^e)$$

(A, c^e) has image of G_{or} is ok. $\Rightarrow c^e \nmid 2$

$$\text{fob. } \forall e, c^e = \varphi_e(f^e(\gamma)) \subset \varphi_e(\underline{g}^{pe} f^e(N)) \subset \underline{g}^{pe} A$$

$\uparrow$
 deg. $\nabla \bar{x}$

(complete $\mathbb{Z}(\bar{x}, \bar{z})$)

$$\nabla \bar{z} \quad c^e \in \bigcap_{e \geq 1} \underline{g}^{pe} A = 0. \quad \checkmark$$

equiv. char $p=0$ a.e.

Def

$$\mathbb{Z}[X, Y] := \mathbb{Z}[X_1, \dots, X_n, Y_1, \dots, Y_m]$$

$E \subset \mathbb{Z}[X, Y]$: finite $\mathbb{Z}$ system of equations e.g.

$$\exists A: \text{ring}, \exists \underline{a} = a_1, \dots, a_n, \underline{b} = b_1, \dots, b_m \in A;$$

$$\forall f \in E, f(\underline{a}, \underline{b}) = 0 \text{ a.e.}, E \nsubseteq A \text{ if } \exists \text{ s.t.}$$

$$\chi \nmid \underline{a} = \underline{a}' \text{ a.e.} \Rightarrow \chi \nmid \underline{b} \text{ if } \chi \nmid \underline{a}'$$

thm (Hochster's meta theorem)

A : Noe. loc. equiv. char. E : sys. of eqn.

E has a solution in A of $\chi \nmid \underline{a}$.

$$\Rightarrow \exists A': \text{Noe. loc. equiv. char } p > 0;$$

E has a solution $\underline{a}', \underline{b}'$ in A' with $\underline{a}'$: s.o.p. of A' .

($A \in \mathbb{F}_p$ -f.g. domain $\in$ maximal ideal $\in$ localise $(\mathbb{F}_p \nmid \chi \nmid \underline{a} \nmid \underline{b})$
 A : regular $\&$ $A' \in$ regular, $\underline{a}' \in$ regular s.o.p. $\chi \nmid \underline{a} \nmid \underline{b}$.)

証明 1.8 Artin's approximation thm 2.5.

Henselisation = $\varinjlim$ etale algebras

$$A^h = \varinjlim (A[x]/f)_{x,m} \quad (f: \text{monic}, a_1 \in m, a_0 \in m)$$

Prop 6

$$1 \leq n, 0 \leq s_1, \dots, s_r \leq n-1, r \geq 2$$

$$\exists m \geq 1; \exists E \subset \mathbb{Z}[X_1, \dots, X_n, Y_1, \dots, Y_m], \text{ s/s of eqns}$$

$$\forall A: \text{ring}, \forall g = a_1, \dots, a_n \in A.$$

$$\exists (M, \alpha) : (s_1, \dots, s_r) - g - \text{modif. of (A.1) degenerate}$$

$$(=) \exists b_1, \dots, b_m \in A : g, b \text{ is solution of } E$$

thm

equiv. char $p=0$ or big $(M \text{ or } \mathbb{F}_p) \nmid \alpha$.

Proof

g : s.o.p. $\forall M \in \text{Mod } A, g$: not M -regular etc.

Prop 6.5) $\exists (M, g) : (s_1, \dots, s_r) - g - \text{modif. of (A.1) degenerate.}$

= a $s_1, \dots, s_r$ i.e. $\nexists$ (Prop 6.5) $\exists E : E$ has a solution in A .

Meta thm 5) $\exists A'$: Noe. loc. equiv. char $p > 0, E$ has a solution in A' .

Prop 6.5) degenerate $\nexists$ g -modif $\nexists$ $\nexists$ Prop 5 & 6 //